Partial Coverage in Homological Sensor Networks

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Abstract—We present a solid study on the performance of a homological sensor network with partial sensing coverage, which means the network has at least one sensing coverage "hole" and we demonstrate that when sacrificing a little coverage the system lifetime can be prolonged significantly. In particular, we showed that when there is one sensing coverage hole (with a coverage rate of 97%) the system lifetime can be extended to 3 − 7 times compared with a full coverage strategy which gives a system lifetime increase with 1.2 − 3 times only.

An algebraic topology tool, homology group, is used in our work to calculate sensing coverage of a sensor network.Unlike other approaches, our method does not need any node location or orientation information and it does not have any assumption about the node deployment control and domain geometry either. The only thing needed to calculate sensing coverage is a node to node communication graph.

I. INTRODUCTION

Recent improvement of micro-electro-mechanical systems (MEMS) technology, digital electronics and wireless communications has a considerable impact on advancing the state of wireless sensor networks (WSNs). We can build low-cost, small size, low-power and multifunctional sensors. Wireless Sensor Networks have emerged as a promising solution for various applications which include health, military, security and smart home [1]–[5]. We firmly believe that in future sensor networks will become an important part of our life.

The power supply to sensor nodes in a network is from battery which can only last for a limited lifetime and make it impossible for a sensor node working for a very long time without recharging. Moreover most sensor network applications are deployed in hostile or remote areas, such as battlefield or forest which make it difficult to replace or recharge node batteries. Therefore, energy conservation operations are critical for extending network lifetime.

Given a set of high density sensor nodes which are over-deployed in a area, our approach is to find several mutually exclusive sub-sets of nodes to work in turn, each of which can maintain a certain degree of sensing coverage (partial or full coverage), meanwhile the remaining sets of nodes are in the sleeping mode. Hence, the overall network lifetime can be increased.

Our work is driven by the following factors. First, it is impossible to manually control the node in a network, so the sensor node has to be able to self-configured. Second, each sensor node does not know its location, since to equip the sensor with GPS device will make sensor node consume much more power and this does not obey the idea of "small size and low-cost node".

Inspired by Robert Ghrist [7], we use an algebraic topology tool (*Homology Group*)to calculate sensing coverage without any information on sensor nodes location and target area coordinate. The algorithm tries to maximize network lifetime and at the same time to maintain the network at some certain coverage rate which is measured by the number of "coverage holes".

Each sensor's sensing area can be approximated as a disk around the sensor. We further assume that each sensor can measure or observe the physical parameter or event in its own sensing area and can use radio-frequency technology to communicate with other sensors in its vicinity. The solution we provided here also assume that a sensor's sensing range r_s is at least larger than $r_b/\sqrt{3}$ where r_b is its brodcasting range. (The reason of this assumption will be explained in section 4)

In the proposed approach, by broadcasting the unique ID, each sensor node will know its neighbors that are within its communication range. Then every node forwards this topology information to the base station. Given the pre-defined number of coverage holes that the sensing system can tolerate, the base station then begins to assign nodes into different working sets, each of which will maintain the required coverage level (k number of coverage holes, where $k \geq 0$). Finally these sets of nodes will be scheduled to work in turn.

Our main aim of the work is to study the performance a homological sensor network when applying partial sensing coverage strategy. By partial coverage, we mean whenever a network has at least one coverage hole.

The contribution of our work is: 1) Use algebraic topology tool to calculate sensing coverage. Unlike other approaches, our algorithm does not need any information on node location. 2) Provide a study of the trade-off between sensing coverage and system lifetime in a homological manner.

We demonstrate that providing high partial sensing coverage rate (1 hole strategy) can significantly increase the lifetime of

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a homological sensor network compared with the network use full sensing coverage strategy.

The rest of paper is structured as follows. The reviews of related work in the literature is presented in section 2. In section 3 we give a brief introduction to simplicial complexes and homology groups. Section 4 describes details of the proposed approach and finally the implementation and simulation results are given in Section 5. Section 6 contains the conclusion and future directions.

II. RELATED WORK

The approaches on calculating sensing coverage can be classified into 3 groups.

The first approach is called computational geometry method [8]–[10]. One feature of this approach is that the precise geometry of the domain and exact locations of the nodes must be known.

The second approach is probabilistic method [11]–[14]. They assume a randomly and uniformly distribute sensor nodes in a domain with a fixed geometry and prove expected area coverage. The main drawback of this method is the need for strict assumptions about the exact shape of the domain as well as the need for a uniform distribution of nodes.

The third approach is called algebraic topology method which uses network topological spaces and their topological invariants. The idea behind this is that the local properties of a sensor network, obtained by local interactions among nodes, can be captured by certain topological spaces. Also the global properties of the sensor network characteristics correspond to certain topological invariants of these spaces. Some attempts from theocratic concepts of computing sensor network coverage have been made in [15], [16]

Furthermore, the work on sensing coverage can be broadly classified in terms of those that provide full coverage [17]–[19] and those provide partial coverage. In full coverage, any point in the sensing area is covered by at least one sensor node. In the application like military surveillance, such full coverage is desired since in a sensitive environment a large number of sensors have to be awake. By contrast, in partial coverage it only requires subset of points in the sensor network are covered and, hence, the number of sensor node awake is reduced.

Up till now, several node scheduling methods [20], [21] have been presented for wireless sensor networks to reduce energy consumption while maintaining sensing coverage at certain desire. All these approaches put a sub-set of sensor nodes into active mode and put rest nodes to sleep. In [20], a node scheduling scheme using off-duty eligibility rule is presented to preserve full sensing coverage. In their work, a sensor node can return to sleep mode only when its neighbors can cover its own sensing area. A simplified approach, sponsored sectors, is used during calculation and this brings node redundancy and energy waste. In [21], individual node can select to enter active mode or sleep mode at the initialization phase. Their approach makes sure that at any given time, the target area is fully covered by sensor nodes. Also, different
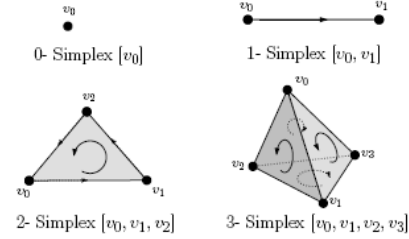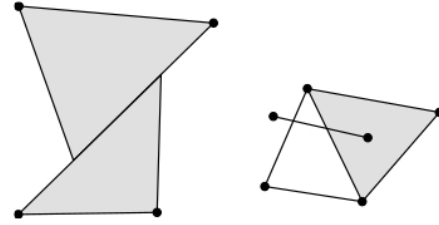

Fig. 1. Simplices example

Fig. 2. Non-examples of simplicial complexes

level of coverage can be achieved by extending (k-coverage $k > 1$) or shrinking the work periods. However, they only worked on 1-coverage (full coverage) and k-coverage (where $k > 1$). All these scheduling approaches need sensor node location or orientation information which is very difficult to get in practical applications and they only work to provide full sensing coverage.

In our work, we put more efforts on partial coverage than full coverage in a homological context which only communication graph is needed during the calculation. Moreover, we expect to combine benefits of full coverage (better surveillance) and partial coverage (longer lifetime). If the sensing coverage rate is relatively high enough, an intruder will always be detected in a short period of time, also a moving intruder will also be detected in a short moving distance. Such partial coverage network is desirable in many applications. A good case is forest fire detection system. In such scenario high partial coverage will ensure that the any fire event will be detected in a reasonably short period of time.

III. SIMPLICIAL COMPLEX AND HOMOLOGY

The mathematical tools being used in our work might not be known by researchers in wireless sensor networks. A brief introduction is presented below. Further readings of various degree of depth can be found at [22]–[24]. In [25] the context of related applications and computations are described.

Simplicial Complex: The topological objects used in our work are called simplicial complexes. Given a set of points V, a k-simplex is an unordered subset $\{v_0, v_1, \cdots, v_k\}$ where $v_i \in V$ and $v_i \neq v_j$ for all $i \neq j$, see Fig 1. The **faces** of a k-simplex consist of all $(k-1)$-simplices of the form $v_0, \ldots, v_{i-1}, v_{i+1}, \ldots, v_k$ for $0 \leq i \leq k$. A **simplicial complex** is a collection of simplices which is closed with

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$$[v_0, v_1, v_2] \xrightarrow{\partial} [v_1, v_2] - [v_0, v_2] + [v_0, v_1]$$

$$[v_0, v_1, v_2, v_3] \xrightarrow{\partial} [v_1, v_2, v_3] - [v_0, v_2, v_3] + \cdots$$
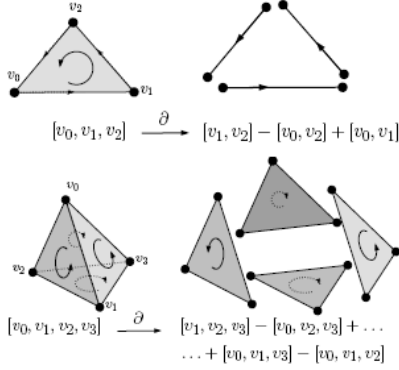$$\cdots + [v_0, v_1, v_3] - [v_0, v_1, v_2]$$

Fig. 3.   The boundary operator on a 2-simplex and 3-simplex

respect inclusion of faces (see Fig 2 for non-examples). A good example is a triangulated surface where vertices of the triangulation correspond to V, edges correspond to 1-simplices, and face correspond to 2-simplices. The orderings of vertices correspond to an orientation.

Simplicial Homology: Homology is an algebraic procedure of counting 'holes' of various types. Let X denote a simplical complex. The homology of X, denoted $H_*(X)$, is a sequence of vector spaces $H_k(X) : k = 0, 1, 2, 3 \cdots$, where $H_k(X)$ is called the k-**dimensional homology of** X. The dimension of $H_k(X)$, called the kth **Betti number of** X, is a coarse measurement of the number of different holes in the space X than can be sensed by using subcomplexes of dimension k.

For example, the dimension of $H_0(X)$ is equal to the number of path-connected components of X and the simplest basis for $H_1(X)$ consists of loops in X, each of which surrounds a different 'hole' in X.

Let X denote a simplicial complex. Define for each $k \geq 0$, the vector space $C_k(X)$ to be the vector space whose basis is the set of oriented k-simplices of X; that is, a k-simplex $\{v_0, \cdots, v_k\}$ together with an order type denoted $[v_0, \cdots, v_k]$ where a change in orientation corresponds to a change in the sign of the coefficient:

$$[v_0, \cdots, v_j, \cdots, v_i, \cdots, v_k] = -[v_0, \cdots, v_i, \cdots, v_j, \cdots, v_k].$$

For k larger than the dimension of X, $C_k(X) = 0$. The **boundary map** is defined to be the linear transformations $\partial : C_k \to C_{k-1}$ which acts on basis elements $[v_0, \cdots, v_k]$ via

$$\partial[v_0, \cdots, v_k] := \sum_{i=0}^{k} (-1)^i [v_0, \cdots, v_{i-1}, v_{i+1}, \cdots, v_k]$$

, as illustrated in Fig 3.

Given the definition of boundary map, we can define a **chain complex**: a sequence of vector spaces and linear transformations

$$\cdots \xrightarrow{\partial} C_{k+1} \xrightarrow{\partial} C_k \xrightarrow{\partial} C_{k-1} \cdots \xrightarrow{\partial} C_1 \xrightarrow{\partial} C_0$$

Consider the following two subspaces of C_k: the **cycles** (those subcomplexes without boundary, which are the kernel

of boundary mapping from vector space C_k to C_{k-1}) and the **boundaries** (those subcomplexes which are themselves boundaries, which are the image of boudnary mapping from vector space C_{k+1} to C_k).

$$k - \text{cycles} : Z_k(X) = \ker(\partial : C_k \to C_{k-1})$$

$$k - \text{boundaries} : B_k(X) = \text{im}(\partial : C_{k+1} \to C_k)$$

It can be easily demonstrated that $\partial \circ \partial = 0$; which means the boundary of a chain has empty boundary. It follows that B_k is a subspace of Z_k. We say that two cycles ξ and η in $Z_k(X)$ are **homologous** if their difference is a boundary:

$$[\xi] = [\eta] \leftrightarrow \xi - \eta \in B_k(X)$$

The k-dimensional **homology** of X, denoted $H_k(X)$ is the quotient vector space,

$$H_k(X) = \frac{Z_k(X)}{B_k(X)}$$

IV. RIPS COMPLEXES AND COVERAGE CALCULATION

In this section, our approach used to calculate sensor network coverage is explained. As mentioned previously, all the information the algorithm acquires is a communication graph.

A. Communication Graphs and Rips Complexes

We first explain the relationship between simplicial complexes and sensor network coverage. As presented in [15], [16], the sensing and communication properties of sensor network can be captured by simplicial complexes and their homological groups. Considering in a wireless sensor network, each identical node has same communication range r and they form a communication graph. In this graph, each vertex stands for a sensor node and an edge between two vertices means that the two nodes are within the communication range r of each other. We now build the rips complex which is generated from communication graph.

Definition IV-A.1. *Given a set of points $v_1, \cdots, v_n$ in $\mathbb{R}^d$ in Euclidean d-space and a fixed radius $r > 0$, the rips complex $\mathcal{R}$, is the simplicial complex whose k-simplices correspond to the unordered $(k + 1)$-tuples of points which are pairwise within a distance r of each other.*

The 0-simplices are the nodes in the communication graph and 1-simplices are all the edge in the graph. Thus, the 2-simplices are the triangle in the graph which has three nodes and each node is within communication range of the other two. The dimension of zero homology group $H_0(\mathcal{R})$ counts the number of connected components of the network. For example, the communication graph is connected if and only if $H_0(\mathcal{R})$ has dimension of 1. The homology group $H_1(\mathcal{R})$ counts the number of the network holes in the communication graph. The hole appears when the part of graph can not be triangulated by any 2-simplices. As shown in [15], [16], these homology groups are related to certain coverage properties of a sensor network.

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In [15], Silva and Ghrist give and prove a theorem listed below which can be used to calculate the network coverage based on some assumptions. The assumptions they have are: First, the identical sensor node has radially symmetric sensing domains r_s which is at least larger than $r_b/\sqrt{3}$ (where r_b is the broadcasting domain of each sensor) and second, the communication graph is connected. Third there are fence nodes which are fixed along the edge of targeted area.

Theorem IV-A.2. *For a set of nodes $\mathcal{X}$ in a domain $\mathcal{D} \subset \mathbb{R}^2$ satisfying those assumptions, the sensors cover whole domain $\mathcal{D}$ if there exits $[\alpha] \in H_2(\mathcal{R},\mathcal{F})$ such that $\partial\alpha \neq 0$. $\mathcal{F}$ is a particular cycle traversing the fence nodes.*

A detailed prove about this theorem is provided in their work [15].

With this theorem we can use communication graph to compute sensing coverage, as long as r_s is at least larger than $r_b/\sqrt{3}$. Hence a sensing coverage of the network can be derived from its communication graph by calculating its homology groups.

B. Coverage Calculation

First we need to get the simplicial complex from the communication graph, which can be easily obtained by asking each node to broadcast its ID. The neighbors within the broadcasting range will pick up the signal and store the sender id.

Simplical complex is calculated in a decentralized way, by two-round broadcasting. After first round, each node will get all 1-simplices memberships and after second round each node will get all 2-simplices memberships. The process of finding 2-simplices memberships is quite straightfoward: each node searches through all its 1-simplices memberships list and tries to find any triangle (3 nodes form a 2-simplices) with 3 vertices which all appear in its 1-simplices memberships list.

This whole process is depicted in Fig 4.

Once all the nodes get their k simplices ($k = 0, 1, 2$) memberships, they will forward their simplices information to the base station which will start to calculate homology groups.

The calculation of homology is by no means of novel and for detailed description of related algorithms please refer to [25]. When, base station gets simplicial complex information, it then starts to calculate $H_0(\mathcal{R})$ and $H_1(\mathcal{R})$. If the betti numbers are 1 and 0 respectively, it means the network has a fully connected communication graph and provides a full sensing coverage as well. On the other hand, if the betti number of $H_1(\mathcal{R})$ is larger than 0, it indicates the area is not fully covered by all the sensor nodes.

Our pre-assumption is that if density of the network is high enough, the monitored area will be fully covered by combination of all the sensors and there will always be some mutually exclusive sets of nodes and each of them will maintain the sensing coverage at a required level (k number of coverage holes, where $k \geq 0$).

The algorithm first computes the minimum number of nodes needed to meet the coverage requirement. And it randomly

(a) Before Broadcast: 0-simplices are discovered.

(b) Before Broadcast: 1-simplices are discovered.

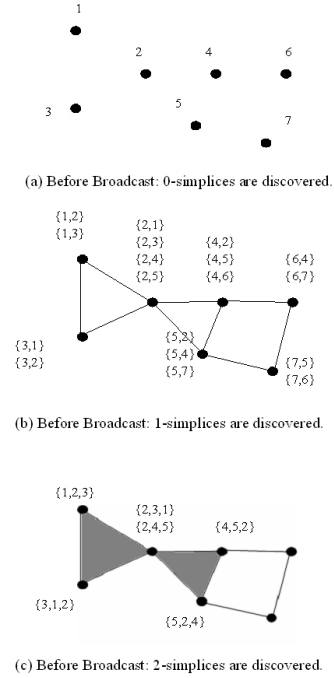(c) Before Broadcast: 2-simplices are discovered.

Fig. 4. The 2-round broadcasting process to calculate simplicial complex

picks this number of nodes and tries to calculate the coverage until the desired coverage level is satisfied. The algorithm has a maximum random picking times n and if after n times trying, the desired requirement is still not reached then the size of picked nodes will be increased by one. Once a set of nodes is found, the set of nodes will be removed and the same picking and calculating process is carried out for the rest of nodes until there is no more node left.

After getting all the sets of nodes, the base station then informs them to work in turn to maximize the overall network lifetime.

Algorithm 1 shows details.

V. EXPERIMENTAL RESULTS

We simulate our algorithm in ns-2 [26] and use Java to provide the visualization. Total number of nodes is set from 20 to 50 with 12 fence nodes circulating the border. The targeted area has a size of 1000 by 1000 and the node communication range is 250. The sensing range of each node is 145 which is larger than $250/\sqrt{3}$. Our approach is the only method by far to carry out sensing coverage calculation without node location information, there is no other simlar approach to be compared with. We run the experiments with different k hole strategies(where $k = 0, 1, 2$).

If there are not enough nodes deployed in the area, there will be a coverage hole in the network. In this case, the betti

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Algorithm 1 Sensing coverage calculation and nodes selection

calculate betti number of H_0 and H_1
if betti number of $H_0 ! = 1$ **then**
  return
else if betti number of $H_1 > 0$ **then**
  display number of holes
else
  $k \leftarrow holeNumber$
  $s \leftarrow startSize$
  $isCover \leftarrow false$
  $n \leftarrow startTryTime$
  while $isCover ==$ false **do**
    set set_s contains randomly picked s nodes
    calculate H_0 and H_1 of set_s
    if $H_0(set_s) == 1$ and $H_1(set_s) == k$ **then**
      $isCover \leftarrow true$
    end if
    $n \leftarrow (n - 1)$
    if $n == 0$ **then**
      $s \leftarrow (s + 1)$
      $n \leftarrow startTryTime$
    end if
  end while
  repeat previous steps with the rest nodes
  schedule the selected sets of nodes to work in turn
end if

Fig. 6.   A network with full sensing coverage

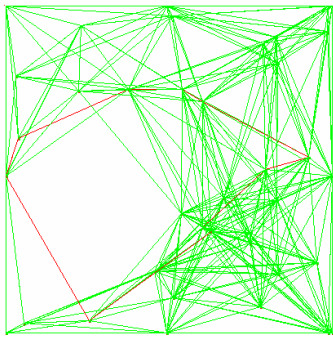Fig. 7.   Number of sensor nodes vs. System lifetime

Fig. 5.   A sensor network with one sensing coverage hole

number of $H_1(\mathcal{R})$ is at least 1, which is shown in Fig 5. The coverage hole ,caused by four nodes at left bottom part of the network, is circled by the red line.

As discussed previously, the algorithm only starts to select sets of nodes when the the sensor network gives a full sensing coverage and the communication graph of the network is fully connected. This requires the betti numbers of $H_0(\mathcal{R})$ and $H_1(\mathcal{R})$ are 0 and 1 respectively, which is depicted in Fig 6.

Through the experiments we found that if the target area only accepts full coverage, then the system lifetime can be extended to 1.2 – 3 times, which only gives a small improvement.

Our main aim of this work is to study the network performance when applying partial sensing coverage strategy. Fig
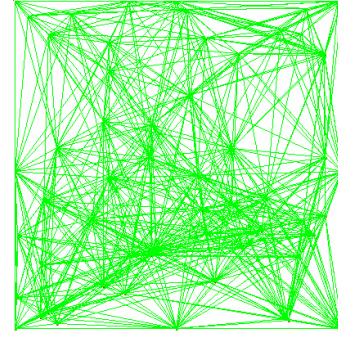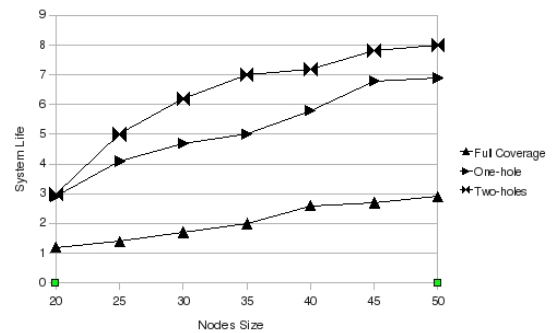
7 illustrates the lifetime increase when different numbers of holes are accepted. We can conclude that when the network uses partial coverage strategy (either 1 or 2 coverage holes), the system lifetime increases significantly. When the system allows 1 coverage hole, its lifetime can be prolonged to 3 – 7 times. If 2 holes are accepted, then the system lifetime can be prolonged from 3 to 8 times. We also notice there is only small increase of system lifetime when changing from 1 to 2 coverage holes.

The average coverage rate for each sub-set of nodes is shown in Fig 8. When system applies 1 coverage hole strategy, any sub-set of nodes, which is in active working mode, can achieve an average coverage rate at 97%. However, this rate is reduced severely to around 80% when 2 holes strategy is used.

A sensor network overall coverage rate is also examined. This overall coverage rate measures how many percentages of the area is covered by combining all the sets of nodes which are scheduled to work in turn. The results have been shown in Fig 9. The overall coverage rates are around 99.5% and 96% respectively when 1 hole and 2 holes strategy are chosen.

This demonstrates that our algorithm can achieve a well balanced hole layout, especially when applying 1 hole strategy. Hence, an intruder can be detected in a short period of time, also a moving intruder will also be detected in a short moving distance.
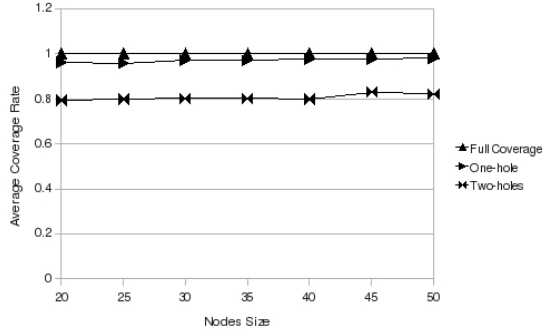
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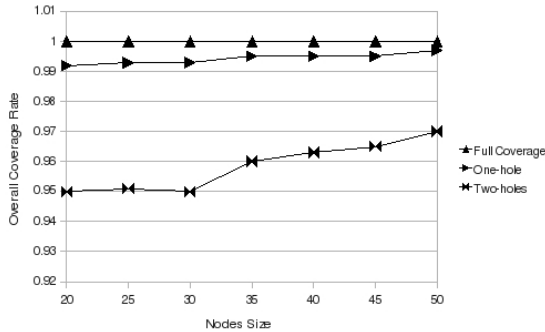Fig. 8. Number of sensor nodes vs. Average coverage

Fig. 9. Overall coverage of a sensor network

VI. CONCLUSION AND FUTURE DIRECTION

An algebraic topology tool is used to calculate sensor network coverage with only communication information. Also a study of the trade-off between sensing coverage and system performance in a homological sensor network is presented. We demonstrate that a homological sensor network, with one sensing coverage hole, can obtain a high level of sensing coverage (97%) while substantially increasing the system lifetime (3 − 7 times) compared with full (100%) sensing coverage strategy only gives 1.2 − 3 times increase. Our algorithm also has a balanced coverage hole layout, which can maintain an overall coverage rate at almost 100%.

However, there are some limitations of our algorithm. Since using topology information only, this makes distributed calculation of homology groups very difficult. By far the calculation of homology group is carried out by the base node. Because of this, the scalability of our system is not good enough. It will take considerably long time to calculate the scheduling scheme when the nodes number exceeds 70.

We also discovered when density of sensor nodes reaches a threshold value (45 nodes in our experiment) the increase of system lifetime will be slow down. This is also due to the centralized algorithm. Since if the total number of the nodes is too large, it will become very difficult to select the minimum number of nodes within n trying times to meet the covering requirement.

In the future work, we will continue to improve our protocol by providing a decentralized algorithm.

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